Risk Sharing, Contagion and Network Formation

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Introduction: Motivation

- Aim to investigate following properties of financial systems:
  - extent of intermediation activity carried out by (financial) firms;
  - effects of different patterns of financial linkages among them, in particular for the capacity of the system to withstand shocks.

  Will *determine the optimal financial systems along these two dimensions*,
  and study whether they can be sustained in *equilibrium*.

- *Key trade-off*: More interconnections

  $\Rightarrow$ higher levels of insurance against shocks hitting each firm, but also

  $\Rightarrow$ higher risk of contagion (large shocks can generate widespread default in the system)

Risk sharing network: features

- Homogeneous firms (banks)
  Linkage - direct or indirect - among two firms: they are in a situation of mutual exposure (to the shocks that may hit each of them)

- Degree of exposure of a firm to another firm: depends on number of linkages of the firm.

- When a random shock hits the income of a firm:
  - all firms linked to the firm hit must bear a part of the shock, increasing in their exposure to the firm
  - firms whose assets' income falls below their liabilities must default, and this is costly

contagion $\Leftarrow$ exposure to common shocks, no counterparty risk ('popcorn' rather than 'domino')

Risk sharing network: main results

- Focus here on network structures which differ along one dimension:

  Degree of *segmentation* (to how many other firms is a firm linked?).

- Characterize the optimal network structure and show how it varies with the probability distribution of the shocks
  Trade-off insurance/contagion clearly emerges:

  a) Optimal segmentation maximal when shock distribution exhibits fat tails, minimal with thin tails;

  b) Intermediate levels of segmentation optimal with sufficient probability mass both on large and small shocks

Risk sharing and intermediation network

- Introduce a second type of firms:
  purchase banks' intermediation services
  $\Longrightarrow$ financial obligations for banks

- Characterize financial system obtained in equilibrium as a result of the bilateral decisions of firms.

- Relative to the optimal system, the equilibrium one features:
  - excessive intermediation activity
  - inefficiently low risk sharing

Literature

- Two main strands:

1. Detailed microfoundation of linkages, simple network and shock structures: Allen and Gale (2000), Freixas et al. (2000), Allen et al. (2011), ...

2. Random (large) networks, simple contagion mechanics: Nier et al. (2007), Leitner (2005), Blume et al. (2011), ...

- More recently: Elliott et al. (2014), Acemoglu et al. (2014), Glasserman and Young (2014)

The Environment

- N (even) financial firms, ex ante identical (for now)
- Each firm i has a risky investment opportunity (a project).
  - with prob. $1-q$, earns normal return R
  - with prob. q, a (random) shock hits: $R-\widetilde{L}_i$.
- Firm i has liabilities on which a - deterministic - gross nominal return M must be paid to investors.
- Shocks are rare: at most one firm is hit by a shock.

The Environment: financial linkages

- **Financial linkages:** each firm i owns fraction a_{ij} of outstanding claims to the yield of firm j's project.

  Financial (network) structure: pattern of linkages among firms, described by $N \times N$ matrix:

$$A = [a_{i,j}]$$

  where $\sum_i a_{ij} = 1$.

- Net financial position of firm i is so:

$$\widetilde{NV}_i = \sum_j a_{ij} \widetilde{R}_i - M$$

The Environment: linkages and default

- When shock L hits return on project of firm i:
  - firm i defaults if $NV_i < 0$, or

$$a_{ii}(R-L) + \sum_{j \neq i} a_{ij} R < M \Leftrightarrow \qquad R - M < a_{ii} L$$

  - firm $j \neq i$ defaults if

$$\left(a_{jj} + \sum_{k \neq i,j} a_{jk} \right) R + a_{ji}(R-L) < M \Leftrightarrow \qquad R - M < a_{ji} L$$

  - Default is costly.
- Linkages generate trade-off between risk sharing and contagion

The Environment: financial linkages

- Financial linkages concern firms' assets, not liabilities.
- Risk of contagion $\Longleftarrow$ firms' exposure to common shocks, because of cross-ownership of claims to their projects.
- Default of a firm has no direct implication for solvency of other firms.

Environment: Welfare

- Will compare the performance of different financial structures. Social welfare is maximal when expected number of firms defaulting is minimal.

- *Here*: Will focus (in this talk) on:
  - symmetric financial structures:

$$a_{ij} = a_{ji} \text{ for all } i, j$$

  - that involve the same *'degree of externalization of risk'* ($a_{ii} = \alpha$ for all i).

Hence they differ only for firms' ability to survive when indirectly (via their linkages) hit by a b shock.

Optimal Financial Structures:

Will consider financial structures that differ in one main dimension:

Degree of segmentation:
describes whether firms are linked to few or many other firms

- will allow for any possible partition of the N firms into disjoint components.

  K_i number of firms to which i is linked to (size of component $K_i + 1$)

- Will determine the pattern of segmentation and of the density of connections which allow to minimize expected defaults in the system, for different kinds of probability distributions of the shocks $\Phi(L_b)$

Optimal Financial Structures: thin/fat tails distributions

Proposition 1. Let L_b be Pareto distributed on $[R - M, \infty)$ $(\Phi(L) = 1 - (R - M/L)^{\gamma})$.

- The optimal degree of segmentation is
i. maximal $(K^* = 1)$ if $0 < \gamma < 1$
ii. minimal $(K^* = N - 1)$ if $\gamma > 1$

When distribution of shocks exhibits fat tails, defaults minimized by minimizing linkages.

With thin tails, optimal to have a single, completely connected component.

Optimal Financial Structures: mixed distributions

Proposition 2. Let $\Phi(L_b)$ be a mixture of two Pareto distributions with $\gamma > 1$ and $\gamma' < 1$, with weights p and $1-p$. Then for an open set of values of p :

- the optimal degree of segmentation is intermediate and the optimal structure is symmetric, with all components of size K^*, $1 < K^* < N-1$.

Risk Sharing and Intermedation

- Introduce *FN* firms of a second type ('*customers*', C), who are potential buyers of *intermediation services* from banks (loans, trade credit, credit lines, ...).

- A bank (B) now may have:
  - linkages with customers (to provide intermediation services, entailing uncertain financial obligations), in return of some payment;
  - linkages with other banks, to share risk (as before).

Risk Sharing and Intermediation

- Payoff of generic B firm (reduced form):

$$u_{B,i} = g\left(n_i^{B,d} + n_i^{B,in}\right) - \alpha n_i^{B,d} - c \cdot \left(n_i^{C,in} + n_i^{C,d}\right) + \sum_{j:(i,j)\in\Gamma} t_j$$

  - $g(.)$ increasing, concave: benefit of risk sharing linkages - direct or indirect - to other B firms (as with thin tail shocks);
  - c : cost of intermediation linkages - direct or indirect . to C firms

Risk Sharing and Intermediation

- Payoff of generic C firm:

$$u_{C,j} = \begin{cases} K_C - cn_j^{C,B,d} - \sum_{i:(i,j)\in\Gamma} t_i, & \text{if } n_j^{B,d} \geq 1 \\ 0, \text{ otherwise} \end{cases}$$

  - benefit of direct linkages to at least one B firm, decreasing in number of other C firms linked to same B

Optimal Financial Structure

- Optimal structure: maximizes total surplus SW in the economy
- At the optimum ach component has a *core periphery structure* where:

  - every C firm is linked to only one B firm

  - B firms are minimally connected among them

  - and each one should be linked to the same number of C types

$$C_B^* = \max\left\{\frac{K_C - (n_B - 1)c}{2c}\right\}$$

- Remains to determine allocation of B types into components: If SW is either everywhere convex or everywhere concave, all components are identical and feature n_B^* firms of type B.

Network formation

- Equilibrium networks, resulting from optimal bilateral contracting choices of individual firms.

  Equilibrium
  (features of bilateral equilibrium (Goyal, Vega Redondo (2007) and pairwise equilibrium (Bloch Jackson 2007):

- Network and collection of transfer payments from C to B firms such that:

  no pair of firms has a profitable deviation, given by:

  - deletion of any subset of their existing linkages *and*
  - formation of a new linkage between the two firms (possibly with the payment of a transfer)

Equilibrium Networks

Proposition 3. In equilibrium:

- each B type is linked to a number

$$\bar{C}_B = \min\left\{F, \frac{K_C - c}{c}\right\}$$

of C types, with $\bar{C}_B > C_B^*$ (*larger* than at the social optimum)

- all components, except one, are identical with a number $\bar{n}_B < n_B^*$ of B types (*smaller* than at the social optimum).

- The remaining component has an even smaller number of B types.

Equilibrium Networks

- Contracting externality in the formation of financial linkages.
- Individual incentives lead to:
  - excessive level of intermediation activity;
  - inefficiently low level of risk sharing and connectivity among B firms.

Conclusions

- Have considered a stylized model to study trade-off of forming linkages between financial firms:
  benefits of risk sharing vs. costs of contagion.

- If shocks are typically large (small), the optimal configuration exhibits maximal (minimal) segmentation in complete components.

- For richer shock patterns, the optimal configuration may involve intermediate levels of segmentation.

- When financial firms also engage in intermediation activity with other firms:
  - optimal network exhibits a core periphery structure
  - optimal structure cannot be obtained as a result of individual linkage formation decisions:
    - excessive intermediation limits extent of risk sharing in the system
    - regulatory interventions could be beneficial.